

LITERAL TERMS

To find the greatest common factor of literal terms, apply either of the two methods used for numerical terms.

Model 1: Find the GCF of x^2y^3 and x^4y .

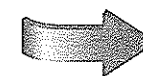
Solution 1: List all the factors. The factors of x^2y^3 are 1, x , x^2 , y , y^2 , y^3 , xy , xy^2 , xy^3 , x^2y , x^2y^2 , and x^2y^3 . The factors of x^4y are 1, x , x^2 , x^3 , x^4 , y , xy , x^2y , x^3y , and x^4y .
The common factors are 1, x , x^2 , y , xy , and x^2y .
 \therefore The GCF of x^2y^3 and x^4y is x^2y .

Solution 2: Use the exponential forms.
The literal terms x^2y^3 and x^4y are in exponential form.
The highest common power of x is x^2 .
The highest common power of y is y^1 .
 \therefore The GCF of x^2y^3 and x^4y is $x^2 \cdot y^1 = x^2y$.

The second solution is preferred for literal terms since they are usually written (or can easily be written) in exponential form.

Model 2: Find the GCF of $aabccc$ and ab^4c^5 .

Solution: First rewrite $aabccc$ as a^2bc^3 .
The highest common power of a is a^1 .
The highest common power of b is b^1 .
The highest common power of c is c^3 .
 \therefore The GCF of a^2bc^3 and ab^4c^5 is $a^1 \cdot b^1 \cdot c^3 = abc^3$.



List all the factors of each literal term.

- 1.26 ab _____
- 1.27 x^3 _____
- 1.28 mnp _____
- 1.29 q^2r _____



Write each literal term in exponential form.

- 1.30 $kkkkkn$ _____
- 1.31 $abcbabc$ _____
- 1.32 $xyyzzz$ _____



Find the GCF of the following sets of literal terms.

- 1.33 x^5y and x^4y^2 _____
- 1.34 $abcde$ and $cdefg$ _____
- 1.35 $m^7n^4p^3$ and $mn^{12}p^5$ _____
- 1.36 $xyyyzz$ and $xxxxzzz$ _____

In general, the greatest common factor of two or more monomials is the product of the GCF of the numerical factors and the GCF of the literal factors.

Model 1: Find the GCF of $8m^3n^2$ and $6m^2n^3$.


Solution: The GCF of 8 and 6 is 2.
The GCF of m^3n^2 and m^2n^3 is m^2n^2 .
 \therefore The GCF of $8m^3n^2$ and $6m^2n^3$ is $2 \cdot m^2n^2 = 2m^2n^2$.

Model 2: Find the GCF of $2abc$, $3bcd$, and $4cde$; then write each monomial as the product of the GCF and the remaining factors of that monomial.

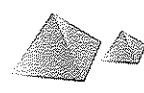
Solution: The GCF of 2, 3, and 4 is 1.
 The GCF of abc , bcd , and cde is c .
 \therefore The GCF of $2abc$, $3bcd$, and $4cde$ is $1 \cdot c = c$.
 The monomials are written $2abc = c(2ab)$,
 $3bcd = c(3bd)$, and $4cde = c(4de)$.

Model 3: Find the GCF of $-9x^2y^7$, $12x^5y^5$, and $-30x^3y^{10}$; then write each monomial as the product of the GCF and the remaining factors of that monomial.

Solution: The GCF of -9, 12, and -30 is 3.
 The GCF of x^2y^7 , x^5y^5 , and x^3y^{10} is x^2y^5 .
 \therefore The GCF of $-9x^2y^7$, $12x^5y^5$, and $-30x^3y^{10}$ is $3x^2y^5$.
 The monomials are written $-9x^2y^7 = 3x^2y^5(-3y^2)$,
 $12x^5y^5 = 3x^2y^5(4x^3)$, and $-30x^3y^{10} = 3x^2y^5(-10xy^5)$.

 Find the GCF of the following sets of monomials.

- 1.37 $5a$, $5b$, and $5c$ _____
- 1.38 $4pq$, $3pq$, and $2pq$ _____
- 1.39 $9x^2y^2$ and $6xy^3$ _____
- 1.40 $-50m^4n^7$ and $40m^2n^{10}$ _____

 Find the GCF of each pair of monomials; then write each monomial as the product of the GCF and the remaining factors of that monomial.

		GCF	PRODUCTS	
Model:	$8abc$ and $-12ac^2$	$4ac$	$4ac(2b)$	$4ac(-3c)$
1.41	$4wxy$ and $6xyz$	_____	_____ ;	_____
1.42	$-10c^2d$ and $15cd^2$	_____	_____ ;	_____
1.43	$38m$ and $57n$	_____	_____ ;	_____
1.44	$12x^5y^9$ and $-35x^7y^3$	_____	_____ ;	_____
1.45	$-80a^3bc^5$ and $-200a^2bc^7$	_____	_____ ;	_____

POLYNOMIALS

The greatest common factor of a polynomial is the GCF of all the terms that make up that polynomial.

Model 1: Find the GCF of $8x^3y^4 - 4x^3y^2 - 6x^2y^2 + 2xy^3$.

Solution: The four terms are $8x^3y^4$, $-4x^3y^2$, $-6x^2y^2$, and $2xy^3$.
 The GCF of 8, -4, -6, and 2 is 2.
 The GCF of x^3y^4 , x^3y^2 , x^2y^2 , and xy^3 is xy^2 .
 \therefore The GCF of the polynomial is $2xy^2$.

The distributive property, $AB + AC = A(B + C)$, is used to factor out or separate the greatest common factor from a polynomial.

Model 2: Factor $10x + 5y$ by separating the GCF.

Solution: The GCF of $10x + 5y$ is 5, and $10x + 5y = 5 \cdot 2x + 5 \cdot y$
 \therefore The factorization is $5(2x + y)$.

Model 3: Factor out the GCF of $9x^3 + 10x^2 - 11x$.

Solution: The GCF is x , and $9x^3 + 10x^2 - 11x = x(9x^2) + x(10x) - x(11)$
 \therefore The factorization is $x(9x^2 + 10x - 11)$.

To check the factorization of a polynomial that results from separating the GCF, two steps should be followed. First, be certain that the GCF of the polynomial in the parentheses is 1. Second, be certain that the original polynomial is obtained when the separated GCF is multiplied by the polynomial in the parentheses.

Model 4: $2(3m - 6mn)$ is *not* the correct factorization of $6m - 12mn$ since the GCF of $3m - 6mn$ is $3m$, not 1. The correct factorization is $6m - 12mn = 6m \cdot 1 - 6m \cdot 2n = 6m(1 - 2n)$.

Model 5: $4(-x + 8)$ is *not* the correct factorization of $-4x + 8$ since $4(-x + 8) = -4x + 32$. The correct factorization is $-4x + 8 = 4(-x) + 4(2) = 4(-x + 2)$

You should note that the binomial $-4x + 8$ will often be factored as $-4(x - 2)$ since $-4x \cdot x$ is $-4x$ and $-4(-2)$ is 8.

Model 6: The polynomial $-10a^3 + 15a^2b - 30ab + 5a$ may be factored as $5a(-2a^2 + 3ab - 6b + 1)$ or preferably as $-5a(2a^2 - 3ab + 6b - 1)$

Model 7: Factor $8x^3y^4 - 4x^3y^2 - 6x^2y^2 + 2xy^3$, and check.

Solution: The GCF of this polynomial is $2xy^2$, and $8x^3y^4 - 4x^3y^2 - 6x^2y^2 + 2xy^3 = 2xy^2 \cdot 4x^2y^2 - 2xy^2 \cdot 2x^2 - 2xy^2 \cdot 3x + 2xy^2 \cdot y = 2xy^2(4x^2y^2 - 2x^2 - 3x + y)$.

Check: The GCF of the polynomial in the parentheses is 1, and the product of $2xy^2$ with each term gives the original polynomial.
 \therefore The factorization is $2xy^2(4x^2y^2 - 2x^2 - 3x + y)$.

 **Factor each polynomial by separating the GCF; check your factors.**

- 1.46 $14a + 7b$ _____
- 1.47 $3y^2 - 4y$ _____
- 1.48 $10x^3 + 8x^2 - 6x$ _____
- 1.49 $4n^4 + n^3$ _____
- 1.50 $-3x - 3y - 3z$ _____
- 1.51 $a^2bc + ab^2c + abc^2$ _____
- 1.52 $5k^2 - 35k^3$ _____
- 1.53 $x^4y^2 + x^3y^3$ _____
- 1.54 $16p^5 - 24p^4$ _____
- 1.55 $-4d^3 + 28d^2 - 4d$ _____
- 1.56 $30y^2z + 12yz^2 - 18yz$ _____
- 1.57 $x^8 + 3x^5$ _____
- 1.58 $2a^5b + 2a^4b^2 + 2a^3b^3$ _____
- 1.59 $48m - 80n$ _____
- 1.60 $-24a^3b^3c^3 - 84a^4b^2c$ _____



Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

SELF TEST 1

Complete these activities (each numbered item, 10 points).

- 1.01 List all the factors (positive integral divisors) of 48.

- 1.02 Write the prime factorization of 7700 in exponential form.

- 1.03 Find the greatest common factor of 270 and 360. (Give your answer in exponential form and in simplified form).
 _____ = _____
- 1.04 List all the factors of mn^2 .

- 1.05 Write $pqqqqrr$ in exponential form.

- 1.06 Find the greatest common factor of $8a^3b^2$ and $12ab^4$.

- 1.07 Write each monomial in Problem 1.06 as the product of the GCF and the remaining factors of the monomial.
 _____ ; _____