Axis of Symmetry and the Vertex from $y = a(x - h)^2 + k$

OBJECTIVE

To find the axis of symmetry and the vertex of a parabola from $y = a(x - h)^2 + k$

From the form $y = a(x - h)^2 + k$, we can identify the axis of symmetry and the vertex of a parabolic function. The identification depends upon transforming $y = ax^2 + bx + c$ into $y = a(x - h)^2 + k$. This process was studied in the previous section.

If $y = a(x - h)^2 + k$, then the following is true:

a. $x - h = 0$ or $x = h$ is the equation of the axis of symmetry. b. (h, k) is the vertex.

Let's find the axis of symmetry and the vertex of $y = x^2 + 6x - 9$.

$$y = x^2 + 6x - 9$$

1. $y + 9 = x^2 + 6x$ — 1. Transpose -9 to the left side of the equation.
2. $y + 9 + \underline{9} = x^2 + 6x + \underline{9}$ — 2. Complete the square. $(\frac{1}{2} \bullet 6)^2 = 3^2 = 9$ Add 9 to both sides of the equation.
3. $y + 18 = (x + 3)^2$ — 3. Factor the right side; combine constants on the left side.
4. $y = (x + 3)^2 - 18$ — 4. Transpose constant term, 18, to the right side.

axis of symmetry

$x - h = 0$ $x + 3 = 0$

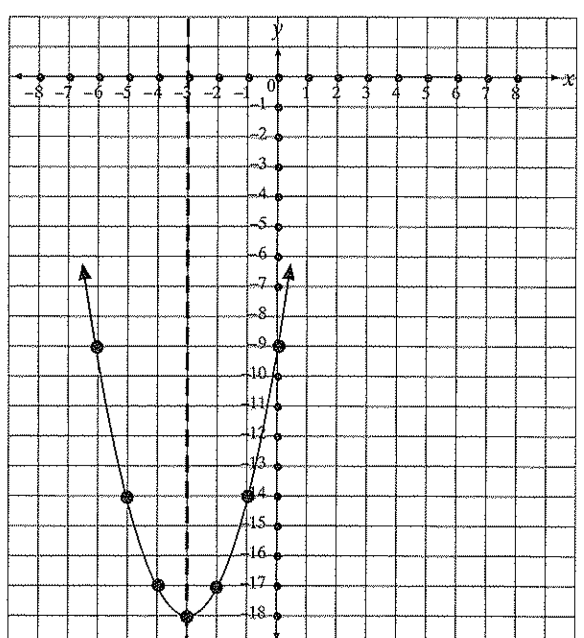
$x = h$ $x = -3$

vertex

(h, k) $h = -3$, $k = -18$

$(-3, -18)$

Let's check our results graphically.



vertex (–3, 18)

axis of symmetry $x = -3$

$y = x^2 + 6x - 9$

x	$x^2 + 6x - 9$	y
–6	36 – 36 – 9	–9
–5	25 – 30 – 9	– 14
–4	16 – 24 – 9	– 17
–3	9 – 18 – 9	– 18
–2	4 – 12 – 9	– 17
–1	1 – 6 – 9	– 14
0	0	– 9

Find the axis of symmetry and the vertex from each parabolic function. (NOTE: If the x^2 term has a negative coefficient, multiply both sides of the equation by -1 as the first step.)

1. $y = x^2 - 10x + 30$ ______________

2. $y = x^2 + 4x - 6$ ______________

3. $y = x^2 + 8x + 10$ ______________

4. $y = x^2 - 2x - 4$ ______________

5. $y = -x^2 + 4x - 5$ ______________

6. $y = -x^2 - 4x + 8$ ______________

7. $y = 2x^2 + 12x + 10$ ______________

8. $y = 3x^2 + 6x - 3$ ______________

9. $y = -4x^2 + 8x + 2$ ______________

10. $y = -5x^2 + 20x - 25$ ______________

After reviewing the Biblical Proverb and Wisdom Principles, respond to these activities.

1. How can you keep your heart pure and beautiful? ______________

2. How valuable is a meek spirit? ______________

3. Meekness is utilizing God's strength for what? ______________

4. Can a meek person be best symbolized as a small match or a great torch? ______________

5. Is meekness the same as weakness? ________ Explain your answer. ______________

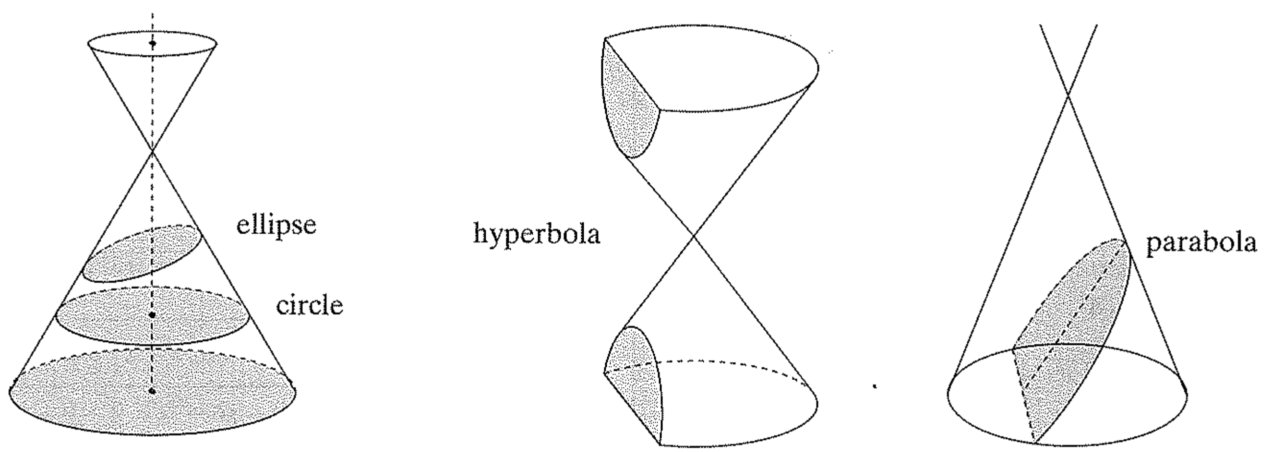
6. When is being great a problem? ______________

7. What is better than being great? ______________

☐ **Score this page.** ☐ **Correct mistakes.** ☐ **Rescore.**

Further Considerations

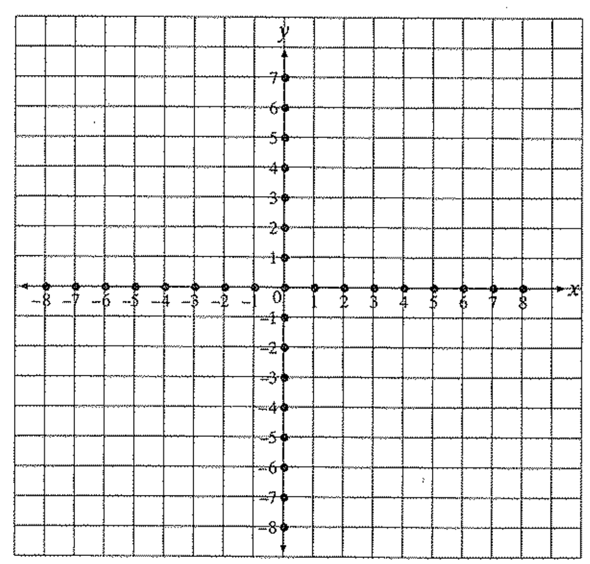
Because mathematics is a field in which our knowledge is continually expanding, its applications also continue to expand. The parabola is one of a family of figures known as conic sections. Conic sections, illustrated below, are formed by planes intersecting cones at different angles.



In your past experience in mathematics, you have already dealt with circles. An ellipse always results from plotting values for an equation having the form $ax^2 + cy^2 = k$. Since graphic solutions for the circle and the ellipse involve use of a square root, we will not demonstrate their solutions here. A hyperbola (hī•pûr′bə•lə) is formed by a plane that intersects two cones attached at their smallest ends. Hyperbolas result from graphing equations having the form $xy = k$. The hyperbola has a solution similar to that of the parabola, provided that $x \neq 0$. (If $x = 0$, y will not be defined.)

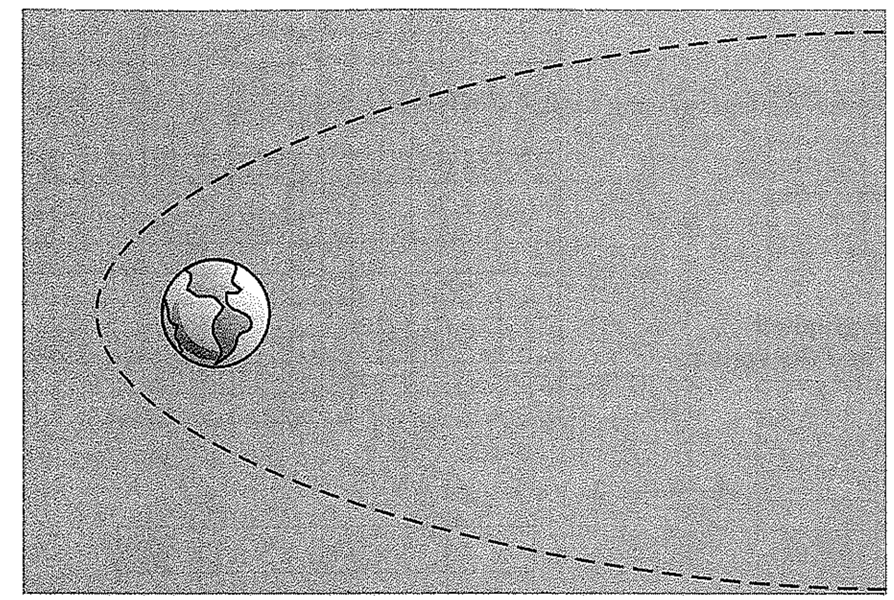
Find the corresponding value if x is given. Using the equation $xy = 4$, plot the resulting points.

x	y
-8	
-4	
-2	
-1	
$-\frac{1}{2}$	
$\frac{1}{2}$	
1	
2	
4	
8	

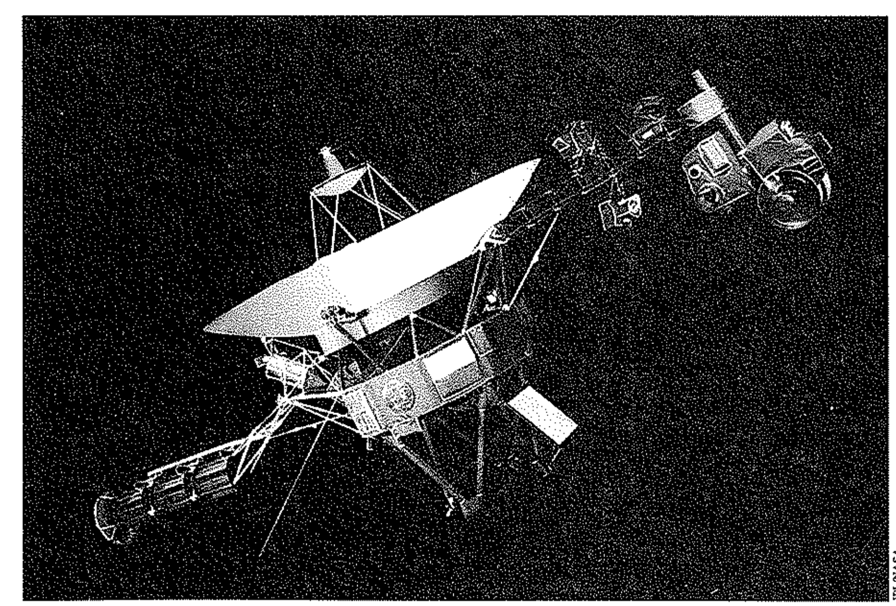


When space probes perform fly-bys of planetary bodies, they approach these bodies in patterns that resemble parabolas. This particular parabola is called a slingshot, or gravity well, maneuver. This trajectory brings the probe close to the planet so observations can be made with an accuracy unobtainable from Earth.

The first picture shows a fly-by pattern of a satellite around a terrestrial body. Notice the parabolic shape of the pattern as the satellite approaches and leaves the outer atmosphere of the body.



Typical slingshot maneuver pattern



Voyager II

A side benefit to this maneuver allows the probe to gain speed and change direction without wasting any fuel. If a probe approaches a planet (as several did during the 1970s) at the proper angle and fast enough to avoid being pulled down by gravity, it will accelerate past the planet and continue on its way in a great curve, faster than when it approached and in the opposite direction from its approach.

In years past, the probes *Pioneer 10* (1972), *Pioneer 11* (1973), *Voyager I* (1977), and *Voyager II* (1977) were all instructed by their ground controllers on Earth to follow parabolas past their targets. *Voyager II* was so uniquely positioned in its course, due to a rare alignment of the outer planets, that the slingshot maneuver was used four times to direct this probe past Jupiter, Saturn, Uranus, and Neptune. After the Neptune fly-by, the probe left the Solar System bound for the stars of deep space.

Mathematics enters our discussion because a slingshot maneuver is a parabola that has its vertex at the point where the probe passes closest to its target. Knowledge of mathematics in general (and algebra in particular) was necessary for ground controllers to guide these four spacecrafts to targets in space under the right conditions for them to carry out their observations.

☐ Score page 34. ☐ Correct mistakes. ☐ Rescore.

CHECKUP

My score ________

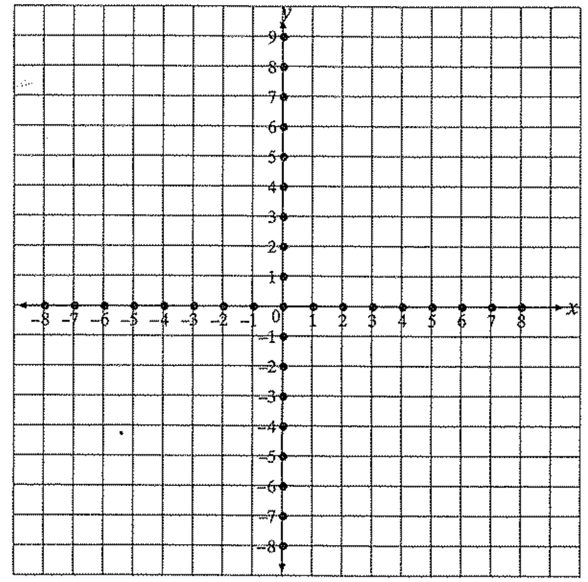
Complete the table, and graph the given quadratic function. (5 points each letter)

1. $y = \frac{1}{4}x^2$

a.

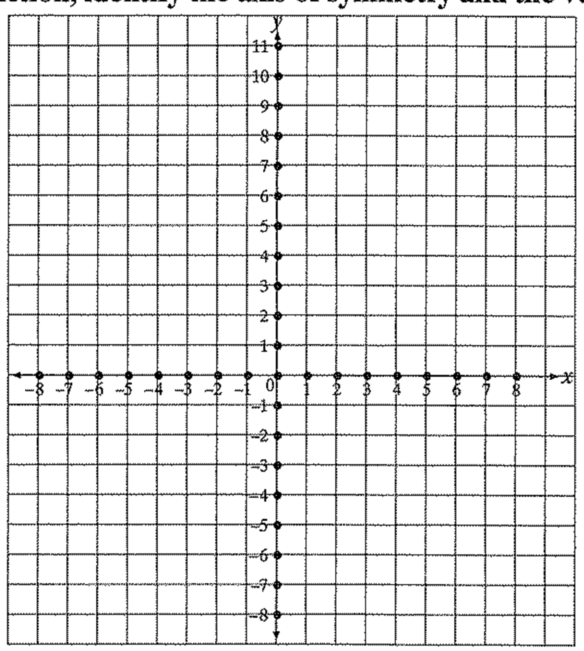
x	x^2	y
−6		
−4		
−2		
0		
2		
4		
6		

b.



Complete each table and graph. On each graphed function, identify the axis of symmetry and the vertex.

b.



2. $y = x^2 - 6x + 5$

a.

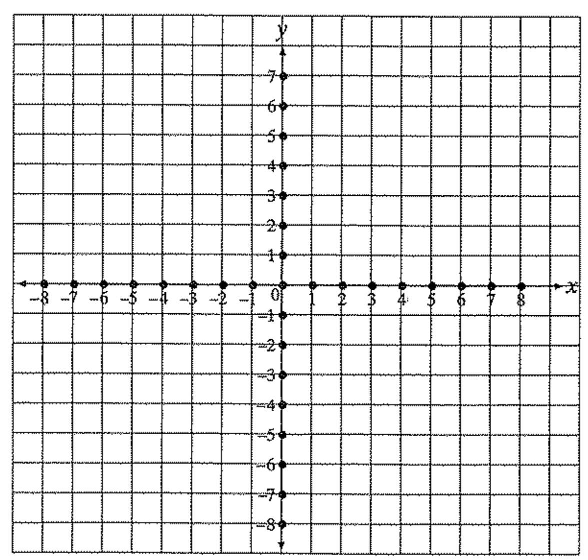
x	$x^2 - 6x + 5$	y
0		
1		
3		
5		
6		

3. $y = 2x^2 + 12x + 15$

a.

x	$2x^2 + 12x + 15$	y
−5		
−4		
−3		
−2		
−1		

b.



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On the blank, write the minimum **or** maximum **point for each function, and give the coordinates for that point.** (You will need to make a table to determine each minimum or maximum point.) (5 points each question)

4. $2x^2 - 20x + 44$ ________
Hint: Start with 3 and increase.

5. $-3x^2 + 12x - 18$ ________
Hint: Start with 0 and increase.

Complete the square in each quadratic function, placing the final answer in the form of $y = a(x - h)^2 + k$.

6. $y = 2x^2 - 12x + 21$ ________

8. $y = 4x^2 - 8x + 6$ ________

7. $y = x^2 + 8x + 15$ ________

9. $y = -3x^2 + 6x - 5$ ________

Find the axis of symmetry and the vertex from each parabolic function. (5 points each answer)

10. $y = 2x^2 - 24x + 22$ ________ ________

12. $y = x^2 - 3x - 5$ ________ ________

11. $y = -x^2 + 2x + 6$ ________ ________

13. $y = 2x^2 - 20x + 44$ ________ ________

☐ **Score pages 36 and 37.** ☐ **Correct mistakes.** ☐ **Rescore.**

STOP!

You must now prepare yourself for the **Self Test.** In your preparation, follow these suggestions:

1. Review the **Objectives.**
2. Review any concepts you still do not understand.
3. Rework every incorrect exercise in this **PACE.**
4. Review the **Checkups.**

When you understand the concepts in this **PACE** and are ready for the **Self Test,** ask your supervisor to initial here. ______

SELF TEST

My score ____________

Give the domain and range of the given relation and equations. (1 point each answer)

1. {(3, 9); (4, –7)} D = ____________ R = ____________
2. $y = 3x$ D = ____________ R = ____________
3. $y = |x|$ D = ____________ R = ____________
4. $x = y$ D = ____________ R = ____________

Determine whether the following relations are functions. (2 points each question)

5. X = {(–1, 4); (4, –1)} ____________
6. Y = {(0, 9); (–4, 9); (2, 9)} ____________

Inverse each given function; then determine whether the result is a function. (1 point each answer)

7. A = {(1, 9); (–3, 2); (5, 9)}
   a. B = ____________
   b. = ____________
8. G = {(6, 7); (7, 6); (3, 2)}
   a. H = ____________
   b. = ____________

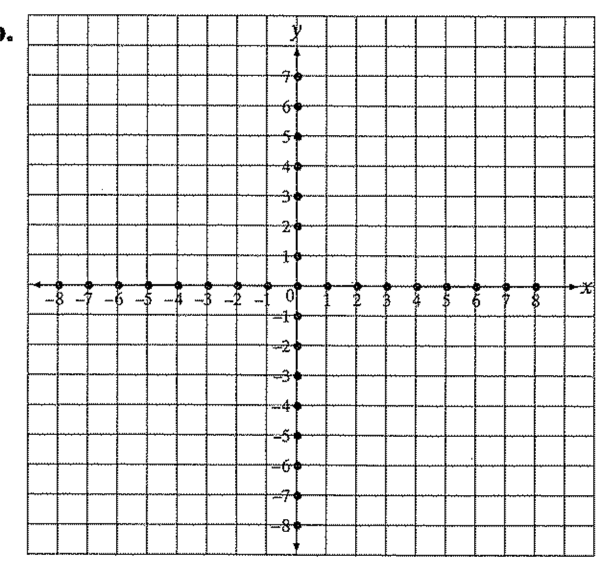
Using the table method, graph the given relation defined by an equation. Determine whether the relation is a function. (2 points each letter)

9. $3x - y = 2$

a.

x	y
0	
1	
2	

b.



c. ____________

Find the slope of the following. (2 points each question)

10. $y = -3x + 4$ ________
11. $2x - 3y - 7 = 0$ ________
12. $x = 9 - y$ ________
13. $-2x = y$ ________
14. $y = 2 - 4x$ ________
15. $3x + 4y = 9$ ________

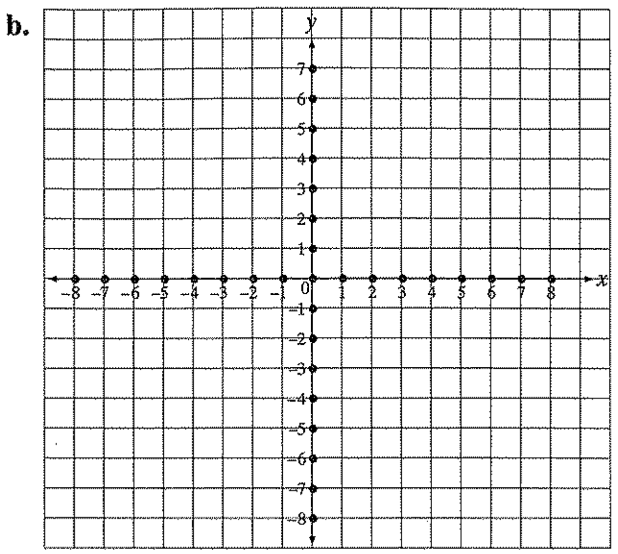
Please continue on the next page.

Find the equation of a relation by using the point-slope or slope-intercept equation.

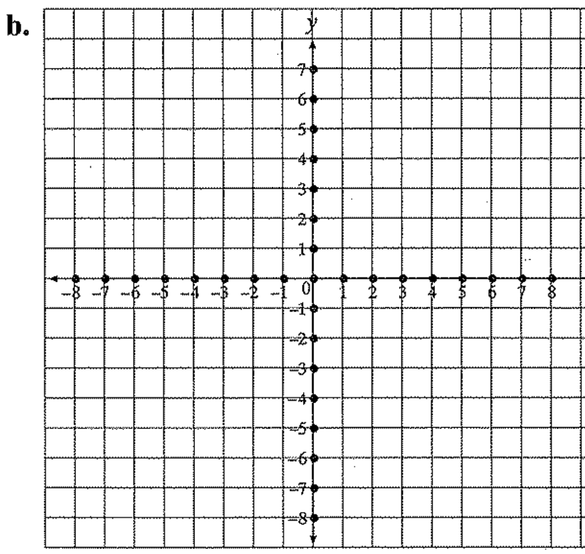
16. $m = \frac{2}{3}$, $(0, -3)$ ____________
17. $m = -\frac{4}{3}$, $(2, -\frac{4}{3})$ ____________

Find the slope and *y*-intercept for the given linear inequalities; then graph the inequalities. (2 points each answer)

18. $2x + y \geq 4$
   a. $m =$ ________, $b =$ ________
   b.

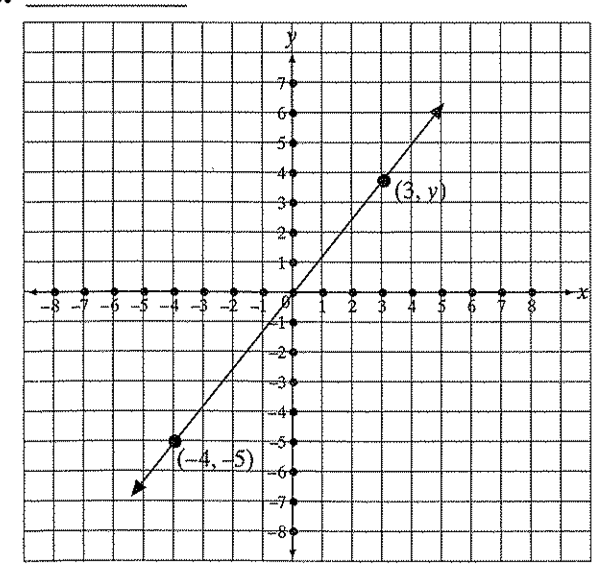


19. $3y - 9 + 2x > -3$
   a. $m =$ ________, $b =$ ________
   b.

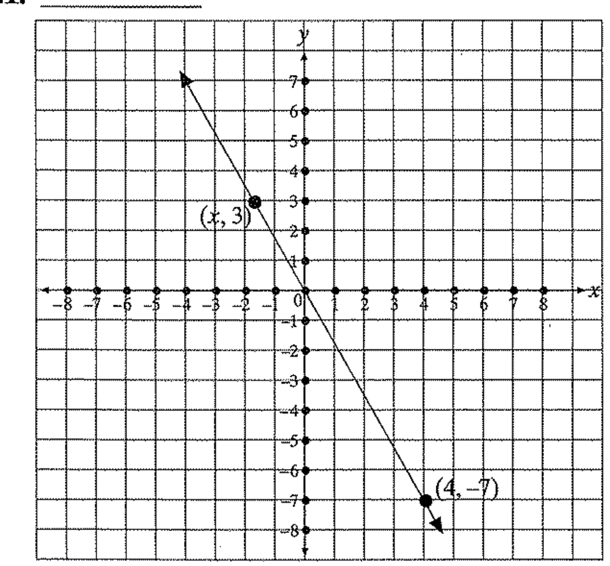


Find the unknown coordinate in the given graphs. (2 points each question)

20. ________



21. ________



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