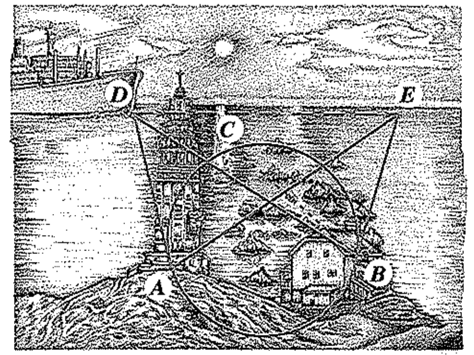
Geometry in Real Life



Areas where rocks and reefs might damage or sink ships are indicated on charts used by ship captains. A lighthouse or landmark will be located at *A* and *B* so angles can be determined. *C* is the point where the ship, on its present course, will enter the "danger zone."

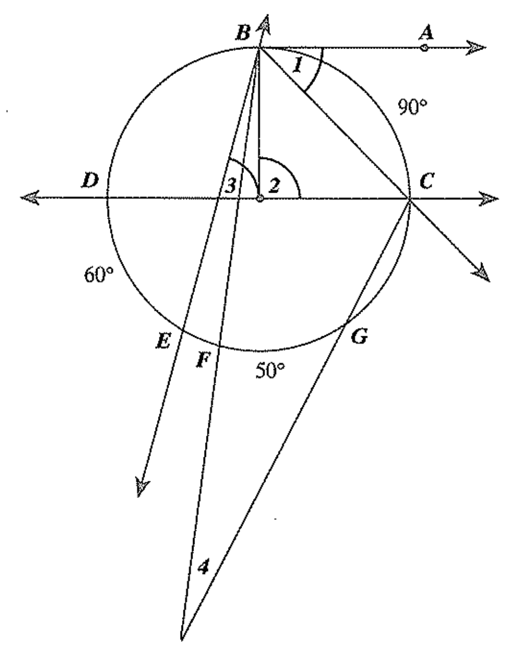
At point *D*, the captain plots an angle smaller than ∠*C*. Then the captain locates a point beyond *C*, equidistant to ∠*D*, and plots another angle, ∠*E*.

By connecting *D* and *E*, the captain has plotted a course that will avoid the rocks and protect his ship from the danger zone.

Study the following examples that show step by step how to solve a problem.

Let's now apply the theorems we have learned. We will find the measurement of each of the numbered angles below.

Given: $m\overset{\frown}{BC} = 90$; $m\overset{\frown}{DE} = 60$; $m\overset{\frown}{FG} = 50$.



Use Theorem 58 to find m∠*1*.

$m\angle 1 = \frac{1}{2} m\overset{\frown}{BC}$

$= \frac{1}{2}(90)$

$m\angle 1 = 45$

Use the definition of measure of minor arc to find m∠*2*.

$m\angle 2 = m\overset{\frown}{BC}$

$m\angle 2 = 90$

Use Theorem 59 to find m∠*3*.

$m\angle 3 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{DE})$

$= \frac{1}{2}(90 + 60)$

$m\angle 3 = 75$

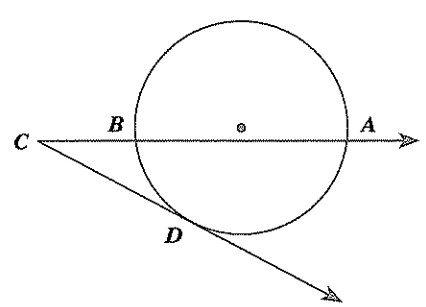
Use Theorem 62 to find m∠*4*.

$m\angle 4 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{FG})$

$= \frac{1}{2}(90 - 50)$

$m\angle 4 = 20$

If $m\overset{\frown}{AD} = 120$ and $m\overset{\frown}{BD} = 35$, then what is m∠*C*?

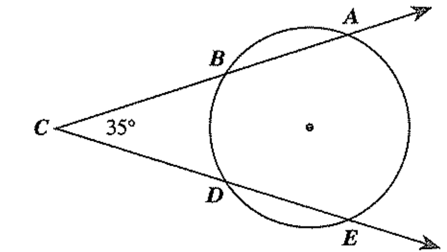


$m\angle C = \frac{1}{2}(m\overset{\frown}{AD} - m\overset{\frown}{BD})$

$= \frac{1}{2}(120 - 35)$

$m\angle C = 42.5$ *or* $42\frac{1}{2}$

If $m\angle C = 35$ and $m\overset{\frown}{BD} = 65$, then what is $m\overset{\frown}{AE}$? (Be sure to set up your equation so that you subtract the small arc from the large arc.)



$m\angle C = \frac{1}{2}(m\overset{\frown}{AE} - m\overset{\frown}{BD})$

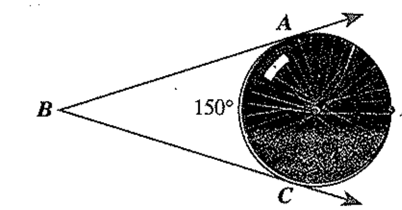
$35 = \frac{1}{2}(m\overset{\frown}{AE} - 65)$

$2(35) = m\overset{\frown}{AE} - 65$

$70 + 65 = m\overset{\frown}{AE}$

$m\overset{\frown}{AE} = 135$

If $m\overset{\frown}{AC} = 150$, then what is $m\overset{\frown}{ADC}$ and m∠*B*? (Use the number of degrees in a circle to find $m\overset{\frown}{ADC}$.)



$m\overset{\frown}{ADC} = 360 - m\overset{\frown}{AC}$

$= 360 - 150$

$m\overset{\frown}{ADC} = 210$

$m\angle B = \frac{1}{2}(210 - 150)$

$m\angle B = 30$

Fill in the blanks.

1. Write Theorem 60. ______________________________

____

____

2. Write Theorem 61. ______________________________

____

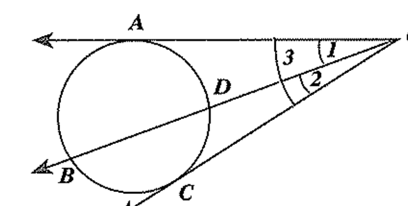
____

3. Write Theorem 62. ______________________________

____

____

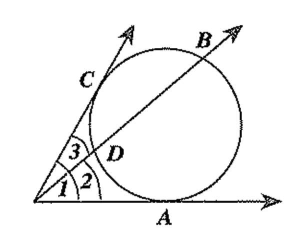
Fill in the blanks. Refer to the preceding examples if you need help.
(You may need separate paper for your calculations.)



4 – 6

4. $m\angle 1 = \frac{1}{2}$(m __________ – m __________)
5. $m\angle 2 = \frac{1}{2}$(m __________ – m __________)
6. $m\angle 3 = \frac{1}{2}$(m __________ – m __________)
7. If $m\overset{\frown}{AC} = 120$ and $m\overset{\frown}{ABC} = 240$, then $m\angle 1$ = __________.
8. If $m\overset{\frown}{AD} = 80$ and $m\overset{\frown}{AB} = 160$, then $m\angle 2$ = __________.
9. If $m\overset{\frown}{BAD} = 240$ and $m\overset{\frown}{CD} = 40$, then $m\overset{\frown}{BC}$ = __________

   and $m\angle 3$ = __________.

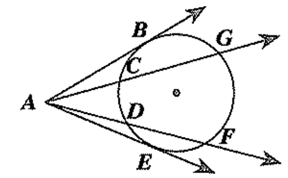


7 – 9

10. If $m\widehat{FG} = 100$ and $m\widehat{CD} = 40$, then $m\angle GAF$ = __________.

11. If $m\angle BAF = 45$ and $m\widehat{BD} = 90$, then $m\widehat{BGF}$ = __________.

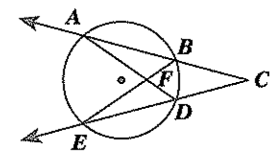
12. If $m\angle FAE = 8$ and $m\widehat{FBE} = 305$, then $m\widehat{DE}$ = __________.



If $m\angle E = 20$ and $m\widehat{AE} = 100$, then:

13. $m\angle C$ = __________.

14. $m\angle AFE$ = __________.



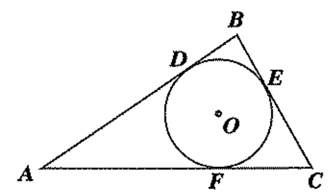
$\triangle ABC$ is circumscribed about $\odot O$; $m\widehat{DE} = 95$, $m\widehat{EF} = 120$, and $m\widehat{DF} = 145$.

15. $m\angle A$ = __________

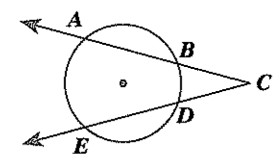
16. $m\angle B$ = __________

Hint: $m\angle A = \frac{1}{2}(m\,\widehat{DEF} - m\widehat{DF})$

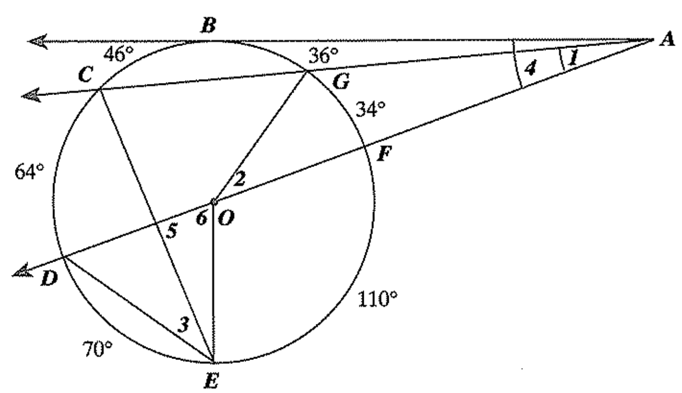
17. $m\angle C$ = __________



18. **CHALLENGE!** If $m\widehat{AE} = 270 - x$ and $m\widehat{BD} = 90 + x$, then $m\angle C$ = __________.



Find the measure of each numbered angle.



Given: $m\widehat{BG} = 36$; $m\widehat{BC} = 46$; $m\widehat{CD} = 64$; $m\widehat{DE} = 70$; $m\widehat{EF} = 110$; $m\widehat{FG} = 34$; $\overline{DF}$ is the diameter.

19. $m\angle 1$ = __________
= __________

20. $m\angle 2$ = __________
= __________

21. $m\angle 3$ = __________
= __________

22. $m\angle 4$ ($m\angle BAF$) = __________
= __________

23. $m\angle 5$ = __________
= __________

24. $m\angle 6$ = __________
= __________

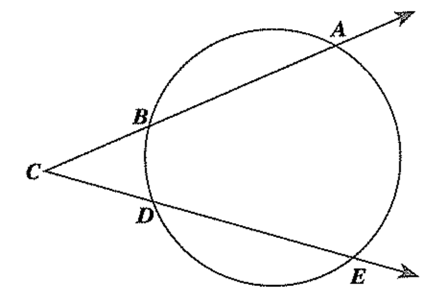
△ Score pages 26 and 27. ◯ Correct mistakes. ▱ Rescore.

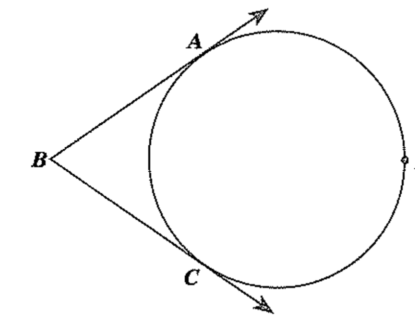
Supervisor initial ________ Please check to see that the student has completed and scored the activities from pages 18, 20, and 24, which were done on separate paper.

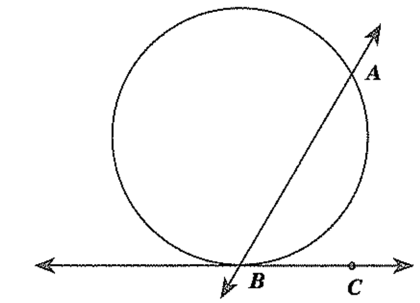
CHECKUP

My score ________
(5 points each question)

Fill in the blanks.



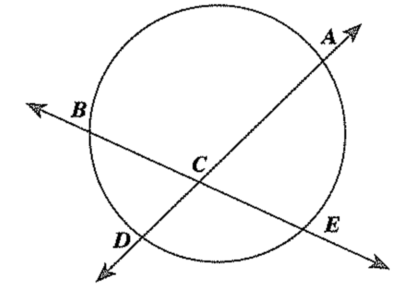


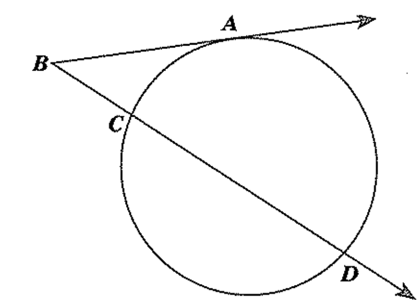


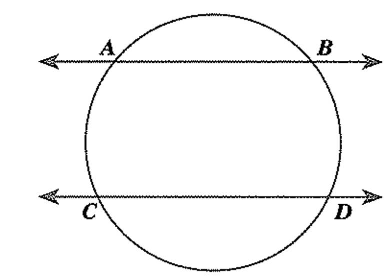
1. $m\angle C$ = __________

2. $m\angle B$ = __________

3. $m\angle ABC$ = __________







$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

4. $m\angle BCD$ = __________

5. $m\angle B$ = __________

6. $m\widehat{AC}$ = __________

Find the measure of each numbered angle. (You may need separate paper for your calculations.)

Given: $m\widehat{AD} = 60$; $m\widehat{DC} = 47$; $m\widehat{DE} = 148$; $m\widehat{FG} = 60$.

7. $m\angle 1$ = __________

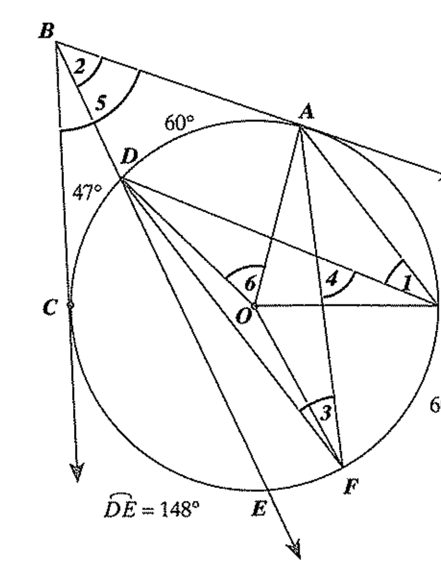
8. $m\angle 2$ = __________

9. $m\angle 3$ ($\angle AFD$) = __________

10. $m\angle 4$ = __________

11. $m\angle 5$ ($\angle ABC$) = __________

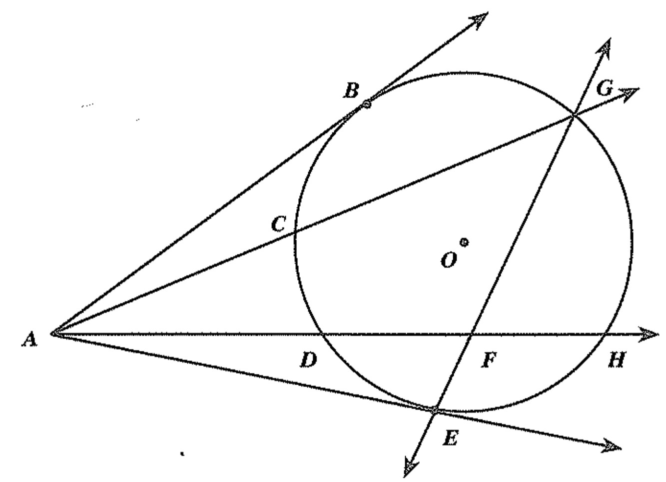
12. $m\angle 6$ ($\angle AOD$) = __________



Please continue on the next page.

Fill in the blanks. (You may need separate paper for your calculations.) (4 points each question)

Given: $\overrightarrow{AB}$ and $\overrightarrow{AE}$ tangent to $\odot O$ at B and E; secants $\overrightarrow{CG}$, $\overrightarrow{DH}$, and $\overleftrightarrow{EG}$.



13. If $m\overset{\frown}{CGE} = 275$, then $m\angle CGE =$ ____________.

14. If $m\overset{\frown}{BE} = 135$, then $m\angle BAE =$ ____________.

15. If $m\overset{\frown}{BC} = 50$ and $m\overset{\frown}{CG} = 125$, then $m\angle BAG =$ ____________.

16. If $m\overset{\frown}{CG} = 125$, $m\overset{\frown}{CD} = 35$, and $m\overset{\frown}{DH} = 120$, then $m\angle GAH =$ ____________.

17. If $m\overset{\frown}{EH} = 70$ and $m\overset{\frown}{DG} = 160$, then $m\angle EFH =$ ____________.

18. The measure of an angle formed by ______________________ that intersect in the exterior of the circle is equal to ____ ______________________ (T. 62)

19. Two ______________________ intersect ______________________ on a circle. (Corollary of Theorem 58)

20. The measure of an angle formed by ______________________ that intersect in the exterior of the circle is equal to ____ ______________________ (T. 60)

21. The measure of an angle formed by a ________________ and a ________________ drawn from the ______________________ is equal to ________________ ______________________________ (T. 58)

22. The measure of an angle formed by ______________________________________ that intersect in the exterior of the circle is equal to ______________________________ ____ (T. 61)

△ Score pages 28 and 29. ◯ Correct mistakes. ▱ Rescore.

Circle Constructions

We now are going to work on some interesting constructions dealing with circles. These constructions will use two or more basic constructions that we learned earlier. Some may look hard, but if you will take it one step at a time, you will succeed. Remember to use only a straightedge and a compass in constructions.

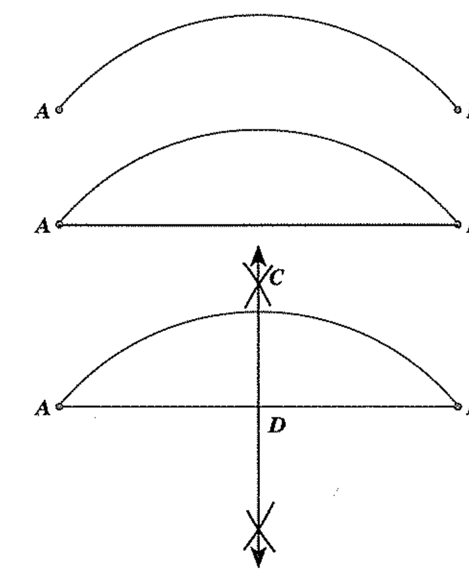
Construct the bisector of an arc.

Construction 1

Given: $\overset{\frown}{AB}$.

Objective: To construct $\overleftrightarrow{CD}$ the bisector of $\overset{\frown}{AB}$.

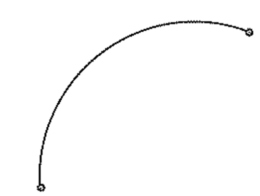
1. Construct chord $\overline{AB}$ of $\overset{\frown}{AB}$ by connecting the endpoints, A and B.
2. Construct $\overleftrightarrow{CD}$ the perpendicular bisector of chord $\overline{AB}$. $\overleftrightarrow{CD}$ also bisects $\overset{\frown}{AB}$.



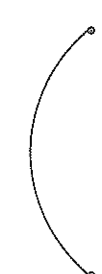
Bisect the following arcs.

1.

3.



2.



4.

Angles Formed by a Tangent and a Secant

In Figure 10, tangent $\overleftrightarrow{AB}$ intersects ⊙O at B, the point of tangency. Secant $\overleftrightarrow{BC}$ (or chord $\overline{BC}$) intersects the tangent at the point of tangency. What is m∠ABC? Use your protractor and find m$\overset{\frown}{BC}$ (m$\overset{\frown}{BC}$ = m∠O). Now measure ∠ABC. Do you find that the m$\overset{\frown}{BC}$ is twice that of m∠ABC? m$\overset{\frown}{BC}$ = 90 and m∠ABC = 45. Theorem 58 is a result of the relationship between an angle formed by a tangent and a secant (or chord) and the intercepted arc.

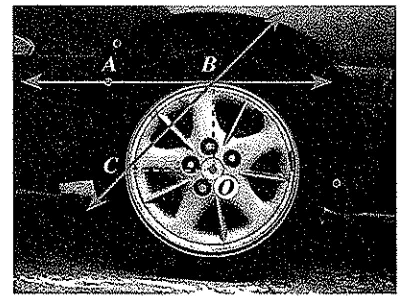


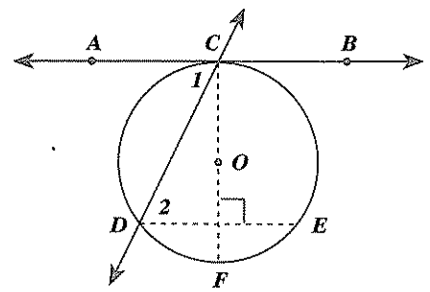
Figure 10

> ***Theorem 58:*** The measure of an angle formed by a tangent and a secant (or chord) drawn from the point of tangency is equal to one-half the measure of the intercepted arc.

Complete this proof of Theorem 58.

Given: $\overleftrightarrow{AB}$ tangent to ⊙O at point C; $\overleftrightarrow{CD}$ is a secant.

Prove: m∠$1 = \frac{1}{2}$m$\overset{\frown}{CD}$.



1. Proof

Statements	Reasons
1. $\overleftrightarrow{AB}$ tangent to ⊙O at point C; $\overleftrightarrow{CD}$ is a secant.	1. Given.
2. Draw $\overline{DE}$ ∥ $\overleftrightarrow{AB}$ through D.	2. A line can be constructed ∥ to a line through a given point not on the line. (T. 28)
3. ______	3. If a trans. intersects two ∥ lines, then the alt. int. ∠s are ≅. (T. 9)
4. m∠1 = m∠2.	4. Definition of ≅ ∠s.
5. ______	5. The measure of an inscribed ∠ is = to $\frac{1}{2}$ the measure of its intercepted arc. (T. 57)
6. Draw diameter $\overline{CF}$.	6. Two points determine a straight line. (P. 1)
7. $\overleftrightarrow{AB}$ ⊥ $\overline{CF}$.	7. If a line is tangent to a ⊙, then it is ⊥ to the radius drawn to the point of tangency. (T. 53) Radii $\overline{OC}$ + $\overline{OF}$ = diameter $\overline{CF}$.
8. $\overline{CF}$ ⊥ $\overline{DE}$.	8. If a line is ⊥ to one of two ∥ lines, then it is ⊥ to the other also. (T. 8)
9. $\overline{CF}$ bisects $\overset{\frown}{DCE}$.	9. If a diameter is ⊥ to a chord, then the diameter bisects the chord and its two arcs. (T. 49)
10. $\overset{\frown}{CD}$ ≅ $\overset{\frown}{CE}$.	10. Definition of bisector of an arc.
11. ______	11. Definition of ≅ arcs.
12. m∠1 = $\frac{1}{2}$m$\overset{\frown}{CE}$.	12. Substitution Property. m∠1 was substituted for m∠2. (Steps 4 and 5)
13. ∴ m∠1 = $\frac{1}{2}$m$\overset{\frown}{CD}$.	13. ______

Using the illustration on the right, study these examples.

If m$\overset{\frown}{BC}$ = 150, find m∠ABC.

m∠ABC = $\frac{1}{2}$m$\overset{\frown}{BC}$

= $\frac{1}{2}$(150)

m∠ABC = 75

If m$\overset{\frown}{BD}$ = 170, find m∠ABD.

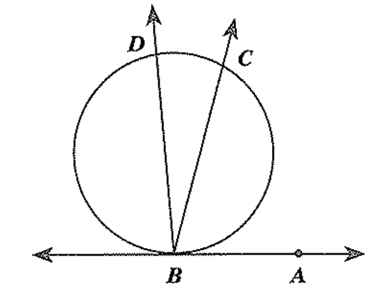
m∠ABD = $\frac{1}{2}$m$\overset{\frown}{BCD}$

= $\frac{1}{2}$(360 − m$\overset{\frown}{BD}$)

= $\frac{1}{2}$(360 − 170)

= $\frac{1}{2}$(190)

m∠ABD = 95



Fill in the blanks. (You may need separate paper for your calculations.)

2. Write Theorem 58. ______________________________

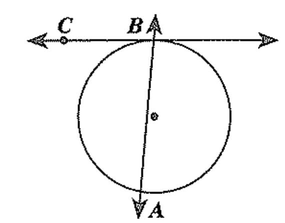
____

____

3. m$\overset{\frown}{AB}$ = 170;

m∠ABC =

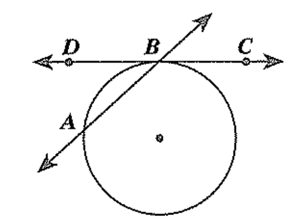
______



5. m$\overset{\frown}{AB}$ = 84;

m∠ABD =

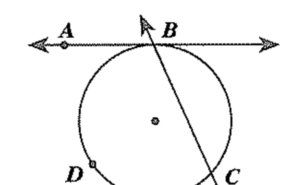
______



4. m$\overset{\frown}{BC}$ = 132;

m∠ABC =

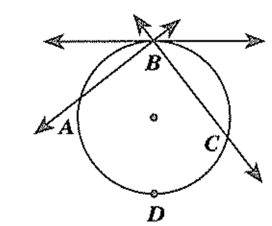
______



6. m$\overset{\frown}{AB}$ = 75;

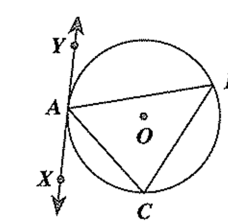
m∠ABC = 90;

m$\overset{\frown}{BC}$ = ______



7. $\overleftrightarrow{XY}$ is tangent to ⊙O at A, m∠BAY = 75, and m∠CAX = 48, what are the measures of the angles of ΔABC?

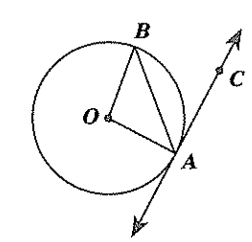
m∠B = ______; m∠C = ______; m∠BAC = ______



On separate paper, write formal proofs.

8. Given: $\overleftrightarrow{AC}$ tangent to ⊙O; $\overline{OA}$ and $\overline{OB}$ are radii.

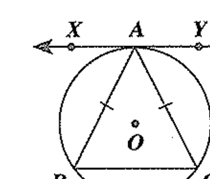
Prove: m∠BAC = $\frac{1}{2}$m∠AOB.



9. Given: ΔABC inscribed in ⊙O; $\overleftrightarrow{XY}$ tangent to ⊙O at A; $\overline{AB}$ ≅ $\overline{AC}$.

Prove: $\overleftrightarrow{XY}$ ∥ $\overline{BC}$.

Hint: Prove ∠BAX ≅ ∠B.



△ **Score pages 17 and 18.** ◯ **Correct mistakes.** ▭ **Rescore.**