If two or more fractions are to be added or subtracted, they must have the same expression or number for their denominators. Keeping that common denominator for the sum or difference, the numerators are then added or subtracted for the numerator of the resulting fraction.

Model 1: $\frac{3}{8} + \frac{1}{8} = \frac{3 + 1}{8} = \frac{4}{8} = \frac{1}{2}$

Model 2: $\frac{5}{3x} + \frac{2}{3x} = \frac{5 + 2}{3x} = \frac{7}{3x}$

Model 3: $\frac{16}{2x - 9} - \frac{7}{2x - 9} = \frac{9}{2x - 9}$

When the denominators are not the same, the LCD must be found and the fractions must be converted to equivalent fractions, as in the previous section.

Model 1: $\frac{5}{8} + \frac{2}{3} - \frac{7}{12}$

$$= \frac{15}{24} + \frac{16}{24} - \frac{14}{24}$$

$$= \frac{15 + 16 - 14}{24}$$

$$= \frac{17}{24}$$

Model 2: $\frac{2a + 7}{4} - \frac{a + 2}{2} + \frac{4a - 3}{5}$

$$= \frac{10a + 35}{20} - \frac{10a + 20}{20} + \frac{16a - 12}{20}$$

$$= \frac{10a + 35 - (10a + 20) + 16a - 12}{20}$$

$$= \frac{10a + 35 - 10a - 20 + 16a - 12}{20}$$

$$= \frac{16a + 3}{20}$$

Note the use of parentheses when subtracting a fraction with more than one term. This step is recommended to avoid a common error in this type of problem.



Model 3:

$$\frac{x - 3}{x^2 - 3x + 2} - \frac{x - 1}{x^2 - 4x + 3} - \frac{x - 2}{x^2 - 5x + 6}$$

$$= \frac{x - 3}{(x - 2)(x - 1)} - \frac{x - 1}{(x - 3)(x - 1)} - \frac{x - 2}{(x - 3)(x - 2)}$$

$$= \frac{(x - 3)(x - 3)}{(x - 2)(x - 1)(x - 3)} - \frac{(x - 1)(x - 2)}{(x - 3)(x - 1)(x - 2)} - \frac{(x - 2)(x - 1)}{(x - 3)(x - 2)(x - 1)}$$

$$= \frac{x^2 - 6x + 9 - (x^2 - 3x + 2) - (x^2 - 3x + 2)}{(x - 3)(x - 1)(x - 2)}$$

$$= \frac{x^2 - 6x + 9 - x^2 + 3x - 2 - x^2 + 3x - 2}{(x - 3)(x - 1)(x - 2)}$$

$$= \frac{-x^2 + 5}{(x - 3)(x - 1)(x - 2)} \text{ or } \frac{5 - x^2}{(x - 3)(x - 1)(x - 2)}$$

Combine as indicated by the signs.

2.31 $\frac{5}{8} - \frac{1}{2}$ __________

2.32 $\frac{4}{9} + \frac{2}{3} - \frac{1}{2}$ __________

2.33 $\frac{3}{a} - \frac{5}{a}$ __________

2.34 $\frac{3a}{4} + \frac{2a}{3} - \frac{a}{12}$ __________

2.35 $\frac{3}{x} + \frac{1}{xyz} + \frac{2}{xy}$ __________

2.36 $\frac{5}{2y} + \frac{1}{6y} - \frac{4}{3y}$ __________

2.37 $\frac{3}{2x^2} - \frac{5}{6xy} + \frac{7}{12y^2}$ __________

2.38 $\frac{8 - y}{3y} + \frac{y + 2}{9y} - \frac{2}{6y}$ __________

2.39 $\frac{4 + a}{a - 3} + \frac{4}{5}$ __________

2.40 $\frac{x - y}{x + y} - \frac{x + y}{x - y}$ __________

2.41 $\frac{x + 2}{x + 4} - \frac{x - 1}{x + 6}$ __________

2.42 $\frac{4}{y^2 - 9} + \frac{5}{y + 3}$ __________

2.43 $\frac{1}{b + 1} + \frac{b^2 + 2}{b^2 - b - 2} - \frac{b}{b - 2}$ __________

2.44 $\frac{a^2 - 5}{a^3 - 1} - \frac{a + 1}{a^2 + a + 1}$ __________

2.45 $\frac{3a - b}{a + 3b} - \frac{a + 2b}{b - 2a}$ __________

2.46 $\frac{y}{y^2 - 16} - \frac{y + 1}{y^2 - 5y + 4}$ __________

2.47 $\frac{x + 6}{x^2 + 8x + 15} + \frac{3x}{x + 5} - \frac{x - 3}{x + 3}$ __________

2.48 $\frac{2x}{y^2 - x^2} - \frac{x}{y - x}$ __________

2.49 $\frac{2x + y}{x + y} - \frac{2x - y}{x - y} - \frac{3x^2 - y^2}{x^2 - y^2}$ __________

2.50 $\frac{2a - 1}{a} - \frac{b + 1}{b} + \frac{a + b}{a + b}$ __________

2.51 $\frac{x - 3}{x^2 - 3x + 2} - \frac{x - 2}{x^2 - 4x + 3} - \frac{x - 1}{x^2 - 5x + 6}$ __________

2.52 $\frac{x}{(x - y)^2} - \frac{2}{x + y} - \frac{x - 3}{x^2 - y^2}$ __________

2.53 $\frac{a}{1 - a^3} - \frac{2}{a^2 + a + 1} + \frac{1}{a^2 - 1}$ __________





In arithmetic a mixed number is any number that consists of a whole number and a fraction, such as $1\frac{2}{3}$. An improper fraction generally can be changed to a mixed number unless the denominator is a factor of the numerator.

Model 1: $\frac{8}{3} = 3\overline{)8}$ → 2 R2 ($8 - 6 = 2$)    $2\frac{2}{3}$ is a mixed number.

Model 2: $\frac{12}{4} = 4\overline{)12}$ → 3    3 is not a mixed number.

An algebraic *mixed expression* is a polynomial in which both fractional and integral terms appear.

> DEFINITION
>
> *Mixed expression:* a polynomial that has at least one integral term and one fractional term.

Model 1: $b + \frac{3x}{y}$

Model 2: $y - 2 + \frac{6ab}{5xy}$

Any mixed expression can be written as an algebraic fraction.

Model 3:
$$b + \frac{3x}{y} = \frac{b}{1} + \frac{3x}{y}$$
$$= \frac{by}{y} + \frac{3x}{y}$$
$$= \frac{by + 3x}{y}$$

The integral terms may be used together for the numerator of a fraction with 1 as the denominator. Using the denominator of the fractional term as the LCD, the two fractions can now be combined as before.

Model 4:
$$y - 2 + \frac{6ab}{5xy} =$$
$$\frac{y - 2}{1} + \frac{6ab}{5xy} =$$
$$\frac{5xy^2 - 10xy}{5xy} + \frac{6ab}{5xy} =$$
$$\frac{5xy^2 - 10xy + 6ab}{5xy}$$

Complex fractions are fractions with a numerator or denominator, or both, that contain fractions.

**DEFINITION**

*Complex fraction:* a fraction with a fraction in its numerator or denominator, or both.

Model 1: $\dfrac{\frac{2}{3}}{7}$

Model 2: $\dfrac{x + \frac{3}{y}}{x - y}$

Model 3: $\dfrac{\frac{2}{3}}{\frac{4}{y}}$

Complex fractions need to be simplified. To do so, find the LCD of the denominators of the fraction or fractions that occur in the numerator or denominator of the original fraction. Then multiply both terms of the original fraction by that number. This process will give you a simplified fraction.

Model 1: $\dfrac{\frac{5}{6}}{\frac{3}{4}}$ The LCD for 6 and 4 is 12.

$$\frac{\frac{5}{6} \cdot \frac{12}{1}}{\frac{3}{4} \cdot \frac{12}{1}} = \frac{10}{9} \text{ or } 1\tfrac{1}{9}$$

Model 2: $\dfrac{\frac{6x^2 - 11x - 35}{9x^2 - 25}}{\frac{2x^2 + 3x - 35}{3x^2 - 5x}}$ The LCD is $9x^3 - 25x$.

$$\frac{\frac{6x^2 - 11x - 35}{1(9x^2 - 25)} \cdot \frac{x(9x^2 - 25)}{1}}{\frac{2x^2 + 3x - 35}{x(3x - 5)} \cdot \frac{x(3x + 5)(3x - 5)}{1}} = \frac{6x^3 - 11x^2 - 35x}{(2x^2 + 3x - 35)(3x + 5)}$$



Model 3: $\dfrac{\frac{a^{-3}}{b^2}}{\frac{b^{-3}}{a^{-2}}}$ The LCD is $a^{-2}b^2$.

$$\frac{\left(\frac{a^{-3}}{b^2}\right)a^{-2}b^2}{\left(\frac{b^{-3}}{a^{-2}}\right)a^{-2}b^2} = \frac{a^{-2}a^{-3}}{b^2b^{-3}} = \frac{a^{-5}}{b^{-1}} = \frac{b}{a^5}$$

Change the following mixed expressions to fractions.

2.54 $7\frac{12}{25}$ ______

2.55 $x + \frac{1}{x}$ ______

2.56 $a - \frac{a}{b}$ ______

2.57 $1 - \frac{2}{3a}$ ______

2.58 $x - \frac{x^2}{x + y}$ ______

2.59 $2m - \frac{1}{m}$ ______

2.60 $a + \frac{1}{a^2}$ ______

2.61 $1 + 2x + \frac{1}{2x}$ ______

2.62 $x - \frac{x^3}{x^2 + y}$ ______

2.63 $2 - \dfrac{x^2 + 10xy + 7y^2}{x^2 + 4xy + 4y^2}$ __________

Change the fractions to mixed expressions.

2.64 $\dfrac{9}{4}$ __________

2.65 $\dfrac{2x - y}{x}$ __________

2.66 $\dfrac{8x^3 - 10x^2}{2x + 1}$ __________

2.67 $\dfrac{x^5 - y^5}{x + y}$ __________

Simplify the complex expressions. Do not leave negative exponents in simplified answers.

2.68 $\dfrac{\frac{a + 1}{a}}{5}$ __________

2.69 $\dfrac{1 + \frac{x}{y}}{1 - \frac{x^2}{y^2}}$ __________

2.70 $\dfrac{\frac{x^2y}{z^2}}{\frac{xy^2}{z^2}}$ __________

2.71 $\dfrac{\frac{a^3 - 27}{a^2 - 9}}{\frac{a^2 + 3a + 9}{a + 3}}$ __________

2.72 $\dfrac{\frac{3x + y}{x - y} - 3}{1 - \frac{x - 3y}{x + y}}$ __________

2.73 $\dfrac{\frac{x^3y^{-2}}{z^2}}{\frac{x^{-2}z^3}{y^{-4}}}$ __________



2.74 $\dfrac{\frac{(x + y)^{-2}}{y^{-3}}}{\frac{x^3y^{-2}(x + y)^{-1}}{x^{-2}}}$ __________

Do the operations as indicated.

2.75 $(\frac{r}{s} + 1) \cdot (\frac{r^2}{s^2 \cdot r^2} + 1)$ __________

2.76 $(\frac{a}{b} - \frac{b}{a})(a - \frac{a^2}{a + b})$ __________

2.77 $(\frac{a}{b} + \frac{1}{a})(a - \frac{a^3}{a^2 + b})$ __________

2.78 $(2a + 5b - \frac{15b^2}{a - b}) \div (2a - \frac{9ab - 15b^2}{a - b})$ __________

2.79 $(\frac{x + y}{x - y} - \frac{x - y}{x + y} + \frac{4y^2}{x^2 - y^2}) \div (\frac{x - y}{x + y} + 1)$ __________

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Review the material in this section in preparation for the Self Test. This Self Test will check your mastery of this particular section as well as your knowledge of the previous section.

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## SELF TEST 2

Complete these items (each answer, 3 points).

2.01 Evaluate when $x = 2$ and $y = 5$: $\dfrac{x^{-2}y^0}{x^3y^{-2}}$. __________

2.02 Write in lowest terms: $\dfrac{a^2 - b^2}{a^3 - b^3}$. __________

2.03 Write in lowest terms: $\dfrac{2x + 4y}{x^2 - y^2} \cdot \dfrac{x + y}{x^2 + 4xy + 4y^2}$.

__________