

Consistent, Inconsistent, and Dependent Equations

OBJECTIVE

To recognize consistent, inconsistent, and dependent equations by graphing

A system of linear equations may be defined as either consistent, inconsistent, or dependent.

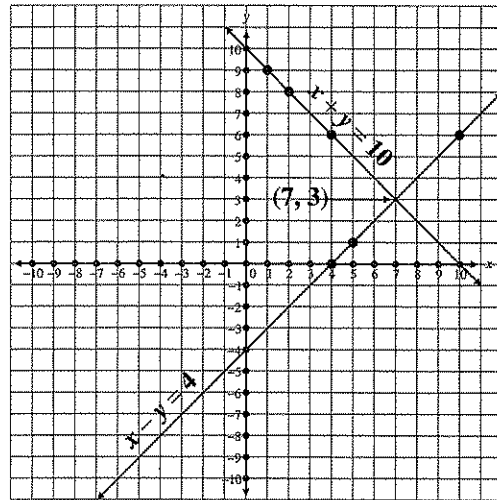
Two linear equations which have only one common solution are *consistent (simultaneous) equations*. Their graphs are two separate lines which intersect at exactly one point.

$$x + y = 10$$

$$x - y = 4$$

$x + y = 10$		$x - y = 4$	
x	y	x	y
1	9	4	0
2	8	5	1
4	6	10	6

Consistent (7, 3)



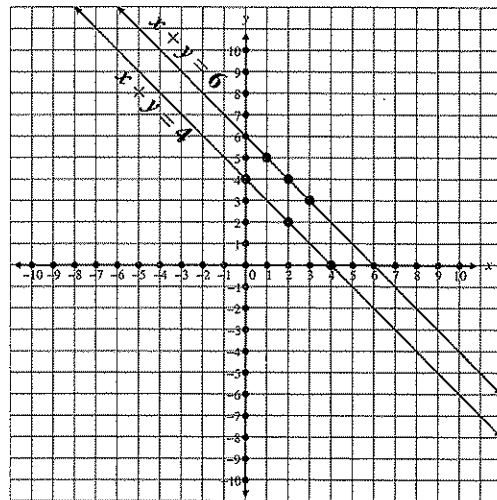
Two linear equations (as in the following example) which have no common solution are called *inconsistent equations*. The graphs of two inconsistent equations are parallel lines.

$$x + y = 6$$

$$x + y = 4$$

$x + y = 6$		$x + y = 4$	
x	y	x	y
2	4	2	2
3	3	0	4
1	5	4	0

Inconsistent



Two *dependent equations* are equivalent linear equations which have exactly the same solution sets. In other words, *all* the solutions for one equation will also satisfy the other equation. The graphs of two dependent equations are identical lines. Notice the example on page 21.

$$x + y = 6$$

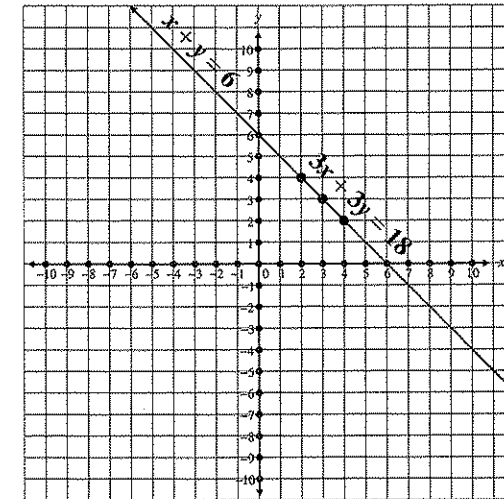
$$3x + 3y = 18$$

$x + y = 6$		$3x + 3y = 18$	
x	y	x	y
2	4	2	4
4	2	4	2
3	3	3	3

Dependent

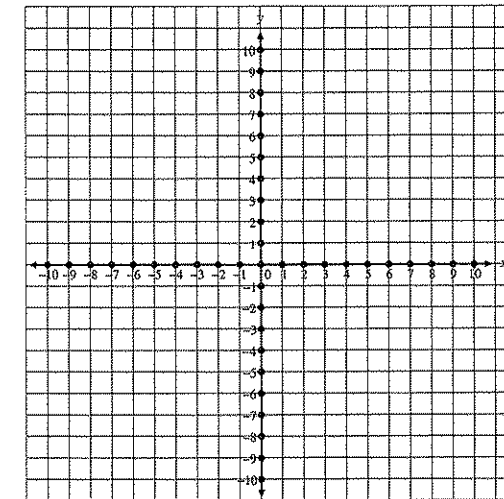
If you study the dependent equations carefully, you will find that they are actually the same equation.

$$\begin{array}{l} x + y = 6 \\ 3(x + y) = 3(6) \\ 3x + 3y = 18 \end{array} \rightarrow 3x + 3y = 18$$

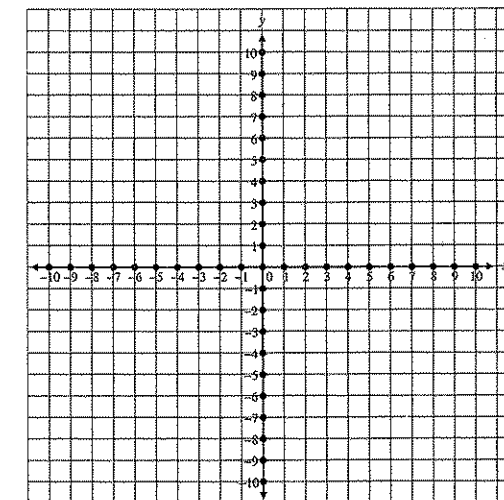


Graph and label the following systems of linear equations. State whether they are consistent, inconsistent, or dependent equations. If they are consistent, give the common solution.

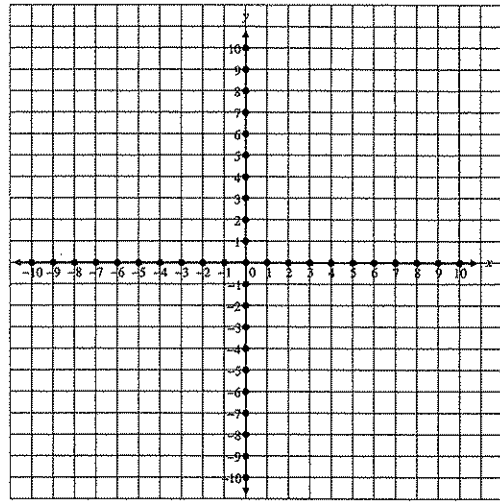
1. $x + y = 5$
 $8x - 2y = 0$



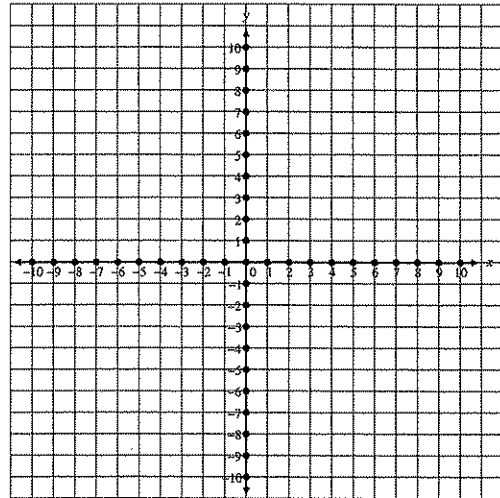
2. $x - y = 4$
 $x - y = -2$



3. $x + y = 4$
 $5x - 2y = -1$



4. $x - 2y = 8$
 $3x - 6y = 24$



If you want to gather honey, don't kick over the beehive.
 Dale Carnegie

Score pages 21 and 22. Correct mistakes. Rescore.

CHECKUP
 (10 points each question)

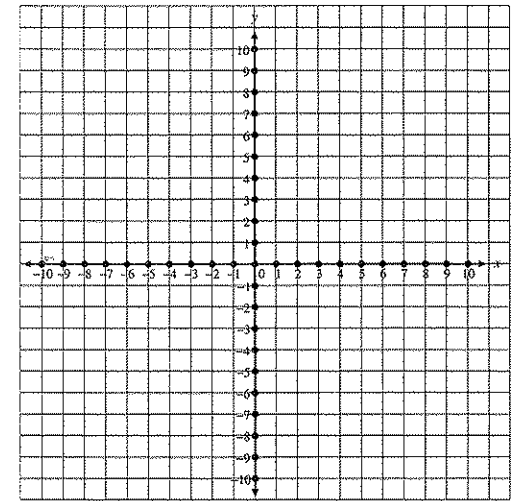
My score _____

Complete these statements.

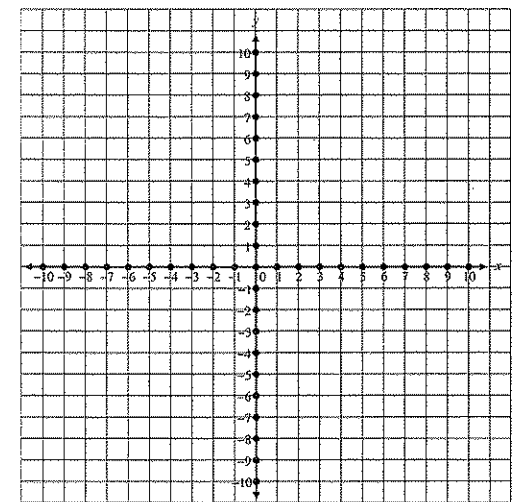
- Linear equations which have only one common solution are _____, or _____, equations.
- Two linear equations which have no common solution are called _____ equations, whose graphs are _____.
- _____ equations are actually the same equation.
- The graphs of two dependent equations are _____ lines.

Graph and label these systems of linear equations. State whether they are consistent, inconsistent, or dependent. If they are consistent, give the common solution.

5. $x - y = -3$
 $x + y = 9$



6. $x - y = -2$
 $x - y = 2$



1. When the equations are combined, the y variable is eliminated.

$$\begin{array}{r} 3x + 3y = 27 \\ 2x - 3y = 3 \\ \hline 5x = 30 \end{array}$$

2. Now, we have a single equation with only one unknown $\rightarrow 5x = 30$. Therefore, we solve for x .

$$\begin{array}{l} 5x = 30 \\ x = 6 \end{array}$$

3. Substitute the value of x , which is 6, into either of the original equations. Solve for y .

$$\begin{array}{l} 3x + 3y = 27 \\ 3(6) + 3y = 27 \\ 18 + 3y = 27 \\ 3y = 9 \\ y = 3 \end{array} \qquad \text{Solution} = (6, 3)$$

4. Check the solution.

$$\begin{array}{l} 3x + 3y = 27 \\ 3(6) + 3(3) \stackrel{?}{=} 27 \\ 18 + 9 \stackrel{?}{=} 27 \\ 27 = 27 \end{array} \qquad \begin{array}{l} 2x - 3y = 3 \\ 2(6) - 3(3) \stackrel{?}{=} 3 \\ 12 - 9 \stackrel{?}{=} 3 \\ 3 = 3 \end{array}$$

STUDY THE FOLLOWING EXAMPLE.

Solve: $-x + y = -4$
 $x + y = -2$

$$\begin{array}{l} 1. \quad -x + y = -4 \\ \quad \quad x + y = -2 \\ \hline \quad \quad 2y = -6 \end{array}$$

Combine the equations.

$$\begin{array}{l} 2. \quad 2y = -6 \\ \quad \quad y = -3 \end{array}$$

Solve for y .

$$\begin{array}{l} 3. \quad -x + y = -4 \\ \quad \quad -x + (-3) = -4 \\ \quad \quad \quad -x - 3 = -4 \\ \quad \quad \quad \quad -x = -1 \\ \quad \quad \quad \quad \quad x = 1 \end{array}$$

Substitute -3 for y into either original equation. Solve for x .

Solution = $(1, -3)$

$$\begin{array}{l} 4. \quad -x + y = -4 \\ \quad \quad -(1) + (-3) \stackrel{?}{=} -4 \\ \quad \quad \quad -1 - 3 \stackrel{?}{=} -4 \\ \quad \quad \quad \quad -4 = -4 \end{array}$$

Check the solution.

$$\begin{array}{l} x + y = -2 \\ (1) + (-3) \stackrel{?}{=} -2 \\ 1 - 3 \stackrel{?}{=} -2 \\ -2 = -2 \end{array}$$

Solve these systems of simultaneous equations by the addition method. Check your work.

1. $x + y = 1$ Solution = _____
 $x - y = -7$

2. $x + y = -1$ Solution = _____
 $x - y = 3$

3. $x + 2y = 35$ Solution = _____
 $3x - 2y = 17$

4. $x + 2y = 12$ Solution = _____
 $x - 2y = 0$

5. $-x + y = 4$ Solution = _____
 $x + 3y = 8$

6. $2x + 2y = 10$ Solution = _____
 $x - 2y = -4$